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NATIONAL INTERACTIVE MATHS OLYMPIAD

NIMO

12 Class | B1 Paper Code

IMPORTANT INFORMATION

1. You are allowed additional 10 minutes to fill the required details in the **RESPONSE SHEET.**
2. The question paper is made as per syllabus guidelines & pattern given in the information Booklet. The Question Paper for Classes 1 to 6 contains 25 Questions each to be answered in 40 minutes. The Question paper for classes 7 to 12 contains 50 Questions each to be answered in 60 minutes. All questions are compulsory. Further instructions are given in the instruction letter to the coordinator teacher.
3. Use the response sheet to mark your responses by darkening the required circle. The response sheet has to be returned to the foundation, duly filled in. The student can retain the Question Paper.

MENTAL ABILITY

1. A wire is in the form of a circle of radius 35 cm. If it is bent into the shape of a rhombus, what is the side of the rhombus?
(1) 32 cm (2) 70 cm (3) 55 cm (4) None of these

2. Which of the following equations has real roots?
(1) $3x^2 + 4x + 5 = 0$ (2) $x^2 + x + 4 = 0$
(3) $(x - 1)(2x - 5) = 0$ (4) None of these

3. In a triangle ABC, $\angle A = 90°$ and D is mid-point of AC. The value of $BC^2 - BD^2$ is equal to
(1) AD^2 (2) $2AD^2$ (3) $3AD^2$ (4) None of these

4. What is the next number in the series 2, 5, 9, 14, 20?
(1) 25 (2) 26 (3) 27 (4) None of these

5. If $\frac{5a+3b}{2a-3b}=\frac{23}{5}$, then the value of $a : b$ is
(1) 2 : 1 (2) 1 : 4 (3) 1 : 2 (4) None of these

6. In a house, there are six 40 watt lamps which are on for 5 hours a day and three 80 watt fans which are on for 10 hours a day. If electricity costs Rs. 2 per kilowatt hour, what is the monthly electricity bill?
(1) Rs. 216 (2) Rs. 280
(3) Rs. 315 (4) None of these

7. A driver's income consists of his salary and tips. During one week his tips were 5/4 of his salary. What fraction of his income came from tips?
(1) 4/9 (2) 5/9 (3) 5/8 (4) None of these

8. What is the value of the following expression?
$(1 + x)(1 + x^2)(1 + x^4)(1 + x^8)(1 - x)$
(1) $1 + x^{16}$ (2) $1 - x^{16}$
(3) $x^{16} - 1$ (4) None of these

9. A worker is paid Rs. 56 for 35-hour week. Upto 40 hours, he is paid at the normal rate and on overtime, 1.5 times the normal. How many hours did he work to get Rs. 88?
(1) 48 (2) 52 (3) 58 (4) None of these

10. In a class, 20 opted for Physics, 17 for Maths, 5 for both and 10 for other subjects. The class contains how many students?
(1) 35 (2) 42 (3) 52 (4) None of these

MATHEMATICS

11. If $x^2 f(x) + f\left(\frac{1}{x}\right) = 0,\ x > 0$ and $I = \int_{1/x}^{x} f(z)\,dz,\ \frac{1}{2} \le x \le 2$, then I is equal to
(1) $f(2) - f(1/2)$ (2) $f(1/2) - f(2)$
(3) 0 (4) none of these

12. If $f'(x) = f(x) + \int_0^1 f(x)\,dx$ and given $f(0) = 1$, then $f(x)$ equals
(1) $\frac{e^x}{2-e}+\frac{1+e}{1-e}$ (2) $\frac{2e^x}{3-e}+\frac{1-e}{3-e}$
(3) $\frac{e^x}{2-e}$ (4) None of these

13. If $f(x) = 2x + 1$, then the number of real values of x for which the three unequal numbers $f(x), f(2x), f(4x)$ are in G.P. is

(1) 1 (2) 2

(3) 0 (4) none of these.

14. If $a_1, a_2, a_3, \ldots$ are in H.P. and $f(k) = \sum_{r=1}^{n} a_r - a_k$,

then $\frac{a_1}{f(1)}, \frac{a_2}{f(2)}, \frac{a_3}{f(3)}, \ldots, \frac{a_n}{f(n)}$ are in

(1) A.P. (2) GP.

(3) H.P. (4) none of these.

15. A sequence of real numbers *i.e.* $a_1, a_2, a_3, \ldots, a_n$ such that $a_1 = 0$, $|a_2| = |a_1 + 1|$, $|a_3| = |a_2 + 1|$, …, $|a_n| = |a_{n-1} + 1|$. Then arithmetic mean $\frac{(a_1 + a_2 + \ldots + a_n)}{n}$ of these numbers cannot be less than

(1) –1/2 (2) –1/3 (3) 1/2 (4) none of these.

16. If α, β are roots of the equation $ax^2 + bx + c = 0$, then the value of determinant $\begin{vmatrix} 1 & \cos(\beta-\alpha) & \cos\alpha \\ \cos(\alpha-\beta) & 1 & \cos\beta \\ \cos\alpha & \cos\beta & 1 \end{vmatrix}$ is

(1) $\sin(\alpha + \beta)$

(2) $\sin\alpha \sin\beta$

(3) $1 + \cos(\alpha + \beta)$

(4) none of these.

17. The number of solutions of $[\sin x] + \cos x = 0$ where $[\cdot]$ denotes the greatest integral function for $x \in [0, 100\pi]$, is

(1) 1 (2) 2 (3) 3 (4) None of these

18. If $f(x)$ is continuous function and attains only rational values and $f(0) = 3$, then roots of equation

$$f(1)x^2 + f(3)x + f(5) = 0 \text{ are}$$

(1) imaginary (2) rational

(3) irrational (4) None of these

19. Suppose $f(x)$ is a quadratic expression positive $\forall$ real x. If $g(x) = f(x) + f'(x) + f''(x)$, then for any real x,

(1) $g(x) < 0$ (2) $g(x) > 0$

(3) $g(x) = 0$ (4) None of these

20. Let $[x]$ denote the greatest integer function. Then in $[0, 3]$, the number of solution of equation $x^2 - 3x + [x] = 0$ is

(1) 6 (2) 4 (3) 2 (4) None of these

21. If a circle having centre at (α, β) radius r completely lies within two lines $x + y = 2$ and $x + y = -2$ then $\min(|\alpha + \beta + 2|, |\alpha + \beta - 2|)$ is

(1) greater than $\sqrt{2}r$

(2) less than $\sqrt{2}r$

(3) greater than $2r$

(4) None of these

22. Portion of asymptote of hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ (between centre and the tangent at the vertex) in the first quadrant is cut by the line $y + \lambda(x - b) = 0$ (λ is a parameter) then

(1) $\lambda \in R$ (2) $\lambda \in (-\infty, 0)$

(3) $\lambda \in (0, \infty)$ (4) none of these.

23. Equation of circle touching the line

$$|x - 2| + |y - 3| = 4 \text{ will be}$$

(1) $(x-2)^2 + (y-3)^2 = 12$

(2) $(x-2)^2 + (y-3)^2 = 4$

(3) $(x-2)^2 + (y-3)^2 = 10$

(4) None of these

24. The focus of rectangular hyperbola

$$(x - h)(y - k) = p^2 \text{ is}$$

(1) $(h - p, k - p)$ (2) $(h - p, k + p)$

(3) $(h + p, k - p)$ (4) none of these.

25. If the ellipse $\frac{x^2}{4} + y^2 = 1$ meets the ellipse $x^2 + \frac{y^2}{a^2} = 1$ in four distinct points and $a = b^2 - 5b + 7$, then b does not lie in

(1) [4, 5] (2) $(-\infty, 2) \cup (3, \infty)$

(3) $(-\infty, 0)$ (4) None of these

26. The equation of the hyperbola referred to its axis as axis of coordinates whose distance between the foci is 16 and eccentricity, $\sqrt{2}$ is

(1) $x^2 - y^2 = 16$ (2) $x^2 - y^2 = 32$

(3) $x^2 - y^2 = 64$ (4) None of these

27. Two adjacent sides of a parallelogram $ABCD$ are $2\hat{i} + 4\hat{j} - 5\hat{k}$ and $\hat{i} + 2\hat{j} + 3\hat{k}$, then the value of $|\overrightarrow{AC} \times \overrightarrow{BD}|$ is equal to

(1) $20\sqrt{5}$ (2) $22\sqrt{5}$

(3) $24\sqrt{5}$ (4) None of these

28. The vector $\vec{a}(x) = \cos x\hat{i} + \sin x\hat{j}$ & $\vec{b}(x) = x\hat{i} + \sin x\hat{j}$ are collinear for

(1) unique value x, $0 < x < \pi/6$

(2) unique value of x, $\pi/6 < x < \pi/3$

(3) no value of x

(4) None of these

29. The equation of plane containing the lines $\vec{r} = \vec{a} + \lambda\vec{b}$ and $\vec{r} = \vec{b} + \mu\vec{a}$

(1) $[\vec{r}\ \vec{a}\ \vec{b}] = 0$

(2) $[\vec{r}\ \vec{a}\ \vec{b}] = \vec{a} \cdot \vec{b}$

(3) $[\vec{r}\ \vec{b}\ \vec{a}] = \vec{a} \cdot \vec{b}$

(4) none of these.

30. If $|\vec{a}| = 2$ and $|\vec{b}| = 3$ and $\vec{a} \cdot \vec{b} = 0$, then $(\vec{a} \times (\vec{a} \times (\vec{a} \times \vec{b}))) =$

(1) $48\vec{b}$ (2) $-48\vec{b}$ (3) $48\vec{a}$ (4) None of these

31. Let a and $b \in R$ such that $0 < a < 1$, $0 < b < 1$. The value of a and b such that the complex numbers $z_1 = -a + i$, $z_2 = -1 + bi$ and $z_3 = 0$ form an equilateral triangle, are

(1) $a = b = 2 - \sqrt{3}$

(2) $a = 2 - \sqrt{3}, b = 2 + \sqrt{3}$

(3) $a = \sqrt{3}$, $b = -\sqrt{3}$

(4) None of these

32. The locus of centre of circle which touches the circles $|z - z_1| = a$ and $|z - z_2| = b$ externally (z_1, z_2 and z are complex numbers) will be

(1) an ellipse (2) a circle

(3) a hyperbola (4) None of these

33. One of the value of i^i is

(1) $e^{-\pi/2}$ (2) $e^{\pi/2}$ (3) e^{π} (4) None of these

34. The number of positive continuous functions $f(x)$ defined in [0, 1] for which $\int_0^1 f(x)dx = 1$, $\int_0^1 xf(x) = a$, $\int_0^1 x^2 f(x)\,dx = a^2$, is

(1) 1 (2) 2 (3) 3 (4) None of these

35. $\int_0^{\pi/3} [\sqrt{3}\tan x]dx = ...$, where $[\cdot]$ denotes greatest integer function

(1) $\frac{5\pi}{6}$

(2) $\frac{5\pi}{6} - \tan^{-1}\left(\frac{2}{\sqrt{3}}\right)$

(3) $\frac{\pi}{2} - \tan^{-1}\left(\frac{2}{\sqrt{3}}\right)$ (4) none of these.

36. If a and b are chosen randomly from the set consisting of first nine natural numbers with replacement, then the probability that the expression $ax^4 + bx^3 + (a+1)x^2 + bx + 1$ has positive values for all real x is

(1) 16/81 (2) 32/81

(3) 64/81 (4) none of these.

37. Two friends Raman and Balwinder have equal number of sons. There are three tickets for a cricket match which are to be distributed among the children of the two friends. The probability that 2 tickets go to the sons of the one and one ticket goes to the sons of the other is 6/7, then the number of children each friend have got is

(1) 6 (2) 4 (3) 3 (4) None of these

38. A teacher takes three children from his class to the zoo at a time as often as he can, but he does not take the same three children to the zoo more than once. He finds that he goes to the zoo 84 times more than a particular child goes to the zoo. Then the number of children in his class is

(1) 15 (2) 20

(3) 60 (4) none of these.

39. Let $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{r}$ be a variable vector such that $\vec{r} \cdot \hat{i}$, $\vec{r} \cdot \hat{j}$ and $\vec{r} \cdot \hat{k}$ are positive integers. If $\vec{r} \cdot \vec{a} \le 12$ then the number of value of $\vec{r}$ is

(1) ${}^{12}C_9 - 4$ (2) ${}^{12}C_9$

(3) ${}^{12}C_4$ (4) none of these.

40. A bag B_1, has m white, n black and p green balls. K_1 balls are randomly transferred to bag B_2 and K_2 balls are randomly transferred to bag B_3. Now one ball is randomly drawn from each bag B_1, B_2 and B_3. Then the probability that all the three balls are of same colour is

(1) $\frac{{}^mC_3 + {}^nC_3 + {}^pC_3}{{}^{(m+n+p)}C_3}$

(2) $\frac{3}{m+n+p}$

(3) $\frac{27}{m+n+p}$

(4) none of these.

ETG INTERACTIVE SECTION

41. If the sum of the distances from two perpendicular lines in a plane is 1, then its locus is :

(1) a square

(2) a circle

(3) a straight line

(4) None of these

42. If the variable takes the values 0, 1, 2,……, n with frequencies proportional to the binomial coefficients $C(n, 1), C(n, 2), ...C(n, n)$ respectively, then the variance of the distribution is :

(1) n (2) $\sqrt{n}/2$

(3) $\frac{n}{2}$ (4) None of these

43. Let A, B and C be three events such that $P(A) = 0.3$, $P(B) = 0.4$, $P(C) = 0.8$, $P(A \cap B) = 0.8$, $P(A \cap B \cap C) = 0.09$. If $P(A \cup B \cup C) \ge 0.75$, then $P(B \cap C)$ satisfies :

(1) $P(B \cap C) \le 0.23$

(2) $P(\mathrm{B} \cap C) \le 0.48$

(3) $0.23 \le P(B \cap C) \le 0.48$

(4) None of these

44. Let ABC be an equilateral triangle formed by weightless inextensible strings, the side AB is horizontal. A and B are tied to fixed points D and E by equal weightness inextensible strings AD, BE; a weight of W gm is attached at C. The angle DAB, ABE are each 150°. Then which of the following statements is true?

(1) The tension in the strings AB, BC and CA are equal

(2) The tensions in the strings BE, AD are inversely proportional

(3) The tension in BE is less than the tension in BC

(4) None of these

45. If the sum of distance of a point from two perpendicular lines is unity. Then its locus is

(1) circle (2) ellipse

(3) hyperbola (4) None of these

46. In a triangle ABC, angle A is greater than angle B. If the values of angles A and B satisfy the equation $3\sin x - 4\sin^3 x - k = 0$; $0 < k < 1$ then the measure of the angle C is

(1) $\pi/3$ (2) $\pi/2$ (3) $2\pi/3$ (4) None of these

47. Three of six vertices of a rectangular hexagon are chosen at random. The possibility that the triangle with three vertices is equilateral, equals

(1) $\frac{1}{2}$ (2) $\frac{1}{5}$ (3) $\frac{1}{10}$ (4) None of these

48. 7 men and 7 women are to sit around a table so that there is a man on either side of a women, the number of seating arrangements is

(1) $(7!)^2$ (2) $(6!)^2$ (3) 6! 7! (4) None of these

49. A circle cut the parabola $(y-3)^2 = 8(x-2)$ at A, B, C, D. A, B, C are feet of the normals drawn from a given point. Then the circle always passes through

(1) $(3, 2)$ (2) $(2, 3)$ (3) $(0, 3)$ (4) None of these

50. If $\vec{a}$, $\vec{b}$, $\vec{c}$ are three non-coplanar vectors, each inclined with other at an angle 30° such that $|a| = |b| = |c| = p$, then volume of tetrahedron whose edges are $\vec{a}$, $\vec{b}$, $\vec{c}$ is

(a) $\left[\frac{3\sqrt{3}+5}{4}\right]^{1/2}\frac{p}{6}$ (b) $\left[\frac{3+5\sqrt{2}}{4}\right]^{1/2}\frac{p}{6}$

(c) $\left[\frac{5\sqrt{2}-3}{4}\right]^{1/2}\frac{p}{6}$ (d) $\left[\frac{3\sqrt{3}-5}{4}\right]^{1/2}\frac{p}{6}$.

END OF THE EXAM