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EDUHEAL FOUNDATION

CLASS 11

LEVEL – 1

Set A1

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EHF NATIONAL INTERACTIVE MATHS OLYMPIAD™

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HMMT

* # See prospectus/website for details

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ROUGH WORK

Instructions for the Candidate

1. You are allowed additional 10 minutes to fill the required details in the RESPONSE SHEET (OMR).
2. The question paper is made as per syllabus guidelines & pattern given in the information Booklet. The Question Paper for Classes 1 to 6 contains 40 Questions each to be answered in 60 minutes. The Question paper for classes 7 to 12 contains 60 Questions each to be answered in 60 minutes. All questions are compulsory. Further instructions are given in the instruction letter to the teacher.
3. Use the response sheet to mark your responses by darkening the required circle. The response sheet has to be returned to the foundation, duly filled in. THE STUDENT CAN RETAIN THE QUESTION PAPER.

MENTAL ABILITY

Directions (Q.1 to Q.4) : Read the following information carefully and answer the questions given below it:

(i) Eight persons E, F, G, H, I, J, K and L are seated around a square table two on each side.

(ii) There are three lady members and they are not seated next to each other.

(iii) J is between L and F.

(iv) G is between I and F.

(v) H, a lady member, is second to the right of J.

(vi) F, a male member is seated opposite E, a lady member.

(vii) There is a lady member between F and I.

1. Who among the following is seated between E and H :

(1) F
(2) I
(3) Cannot be determined
(4) None of these

2. How many persons are seated between K and F :

(1) One (2) Two
(3) Three (4) Cannot be determined

3. Who among the following are the three lady members:

(1) E, G and J (2) E, H and G
(3) G, H and J (4) Cannot be determined

4. Who among the following is to the immediate left of F:

(1) G (2) I
(3) J (4) Cannot be determine

Direction (Q. 5 to Q. 7) :

Refer to the data below and answer the questions that follow:

In a survey of 1000 households, washing machines, vacuum cleaners, and refrigerators were counted. Each house had at least one of these appliances, 400 had no refrigerator, 380 no vacuum cleaners, and 542 no washing machines. 294 had both a vacuum cleaner and a washing machine, 277 both a refrigerator and a vacuum cleaner, 120 both a refrigerator and a washing machine.

5. How many had only vacuum cleaner?

(1) 112 (2) 123
(3) 62 (4) 138

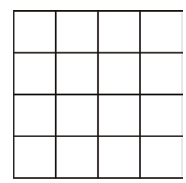
6. How many had at least two of the three appliance?

(1) 665 (2) 652
(3) 529 (4) None of these

7. How many had exactly one appliances?

(1) 550 (2) 335
(3) 216 (4) 500

8. How many rectangles are there in the following figure?



(1) 34 (2) 100
(3) 98 (4) 66

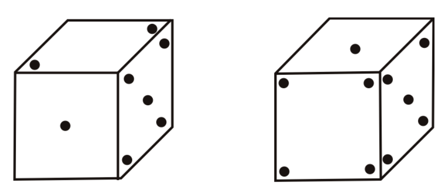
9. Take the given statements as true and decide which of the conclusions logically follow from the statements

Statements: All the cats are rats.
Some dogs are rats.

Conclusions: (I) All the cats are dogs.
(II) Some dogs are cats.

(1) Only (I) conclusion follows
(2) Only (II) conclusion follow
(3) Neither (I) nor (II) follows
(4) Either (I) or (II) follows

10. Observe the dots on a dice (one to six dots) in the following figures. How many dots are contained on the face opposite to that containing four dots?



(i) (ii)

(1) 2 (2) 3
(3) 6 (4) Cannot be determined

MATHEMATICS

11. Let A and B be two sets such that $A \cup B = A$. Then $A \cap B$Then is equal to

(1) ϕ (2) A

(3) B (4) none of these

12. For $0 < \phi < \frac{\pi}{2}$, if $x = \sum_{n=0}^{\infty} \cos^{2n} \varphi, y = \sum_{n=0}^{\infty} \sin^{2n} \varphi$ and $z = \sum_{n=0}^{\infty} \cos^{2n} \varphi \sin^{2n} \varphi$, then

(1) $xyz = x^2 + y^2 + z^2$ (2) $xyz = xy + z$

(3) $xyz = xy + yz + zx$ (4) none of these

13. If $A = \{x : x \in I, x^4 - x^3 - 2x^2 + 2x = 0\}$ and $B = \{x : x \in N, 2x^2 - 1 < 7\}$ then

(1) $A = B$

(2) $A \subset B$

(3) A and B aren't comparable

(4) $B \subset A$

14. In a group of 900 people, there are 650 who can speak kannada and 400 who can speak Hindi. How many speak both Kannada and Hindi?

(1) 100 (2) 150

(3) 200 (4) 300

15. If $A \cap B = A$ and $B \cap C = B$, then $A \cap C$ equals

(1) C (2) B

(3) A (4) $B \cup C$

16. If A, B and C are three sets, then $A \cap (B - C) =$

(1) $(A - B) \cap (A - C)$ (2) $A - (B \cap C)$

(3) $(A \cap B) - (A \cap C)$ (4) $(A \cap B) \cup (A \cap C)$

17. The value of $\frac{\cos 12^o - \sin 12^o}{\cos 12^o + \sin 12^o} + \frac{\sin 147^o}{\cos 147^o} =$

(1) 1 (2) -1

(3) 0 (4) none of these

18. If $\cos\theta = \cos\alpha \cos\beta$ then $\tan\frac{\theta+\alpha}{2}\tan\frac{\theta-\alpha}{2}$ is equal to

(1) $\tan^2\frac{\alpha}{2}$ (2) $\tan^2\frac{\beta}{2}$

(3) $\tan^2\frac{\theta}{2}$ (4) $\cot^2\frac{\beta}{2}$

19. The value of $3(\sin^4\theta + \cos^4\theta) - 2(\sin^6\theta + \cos^6\theta)$ is

(1) 0 (2) –1

(3) 1 (4) 2

20. If $\frac{2\sin\alpha}{1+\cos\alpha+\sin\alpha} = y$, then the value of $\frac{1-\cos\alpha+\sin\alpha}{1+\sin\alpha}$ is

(1) sin α (2) cos α

(3) y (4) $\frac{y}{2}$

21. Solve :

$$\frac{(x+1)(2x+3)^2(x^2+x+1)}{(x-1)^4(x+7)} \geq 0$$

(1) $(-\infty, -7) \cup [-1, 1) \cup (1, \infty) \cup \{-\frac{3}{2}\}$

(2) $(-\infty, -7) \cup [-1, 1] \cup [1, \infty) \cup \left[-\frac{3}{2}\right]$

(3) $(-\infty, -7) \cup (-1, 1] \cup (1, \infty)$

(4) $(-\infty, -7] \cup (-1, 1] \cap \{1, \infty\}$

22. If $x^2 + 4x + a \geq 0$ for all x , then a lies in

(1) $(-\infty, 2)$

(2) $[2, 4]$

(3) $[4, \infty)$

(4) $(\infty, 4)$

23. Solve for 'x'

$$\frac{(x+1)^2(x-2)(x-3)^2(x-4)^4}{(x-5)(x-6)^2(x-7)} \le 0$$

(1) $(-\infty, 2)\cup(8, 6)\cup(6, 7)$

(2) $(-\infty, 2)\cup(6, 7)\cup\{3, 4\}$

(3) $(-\infty, 2]\cup\{5, 6\}\cup(6, 7)\cup[3, 4]$

(4) $(-\infty, 2]\cup(5, 6)\cup(6, 7)\cup\{3, 4\}$

24. The domain of the function $f(x)=\sqrt{\frac{x(x-1)}{x+2}}$ is

(1) $(-2,0]\cup[1,\infty)$ (2) $(-2,0)\cap\{1,\infty\}$

(3) $[1,\infty)$ (4) $(-2,0]$

25. In a ΔPQR, $R=\frac{\pi}{2}$. If $\tan\left(\frac{P}{2}\right)$ and $\tan\left(\frac{Q}{2}\right)$ are the roots of $ax^2+bx+c=0, a\ne 0$, then

(1) b = c (2) b = a + c

(3) a = b + c (4) c = a + b

26. The domain of $f(x) = \log_2 \log_3 \log_{\frac{4}{\pi}}\left(\tan^{-1}x\right)^{-1}$ is

(1) (–1, 1) (2) (0, 1)

(3) $\left(\frac{4}{\pi},\infty\right)$ (4) R

27. The domain of $f(x)=\frac{1}{|\sin x|+\sin x}$ is

(1) R

(2) $\bigcup_{n\in Z}\left((2n+1)\pi, 2(n+1)\pi\right)$

(3) $\bigcup_{n\in Z}\left(2n\pi,(2n+1)\pi\right)$

(4) ϕ

28. The value of (x + y) (x – y) + (x + y) (x – y) (x^2+y^2) + $\frac{1}{3!}$ (x + y) (x – y) $(x^4+y^4+x^2y^2)$+......

(1) $e^{x^2}+e^{y^2}$ (2) $e^{x^2}-e^{y^2}$

(3) $e^{x^2-y^2}$ (4) None

29. $y=x-\frac{x^2}{2}+\frac{x^3}{3}-....$ then x equal to

(1) log (1 + y) (2) $e^y - 1$

(3) $y+\frac{y^2}{2!}+\frac{y^3}{3!}+....$ (4) $e^{-y} - 1$

30. Of the number of three athletic teams in a school, 21 are in the basketball team, 26 in hockey team and 29 in the football team, 14 play hockey and basketball, 15 play hockey and football 12 play football and basketball and 8 play all the games. The total number of members is:

(1) 42 (2) 43

(3) 45 (4) none of these

31. Find the domain of f(x) = $\frac{x^2-3x+4}{x^2+x+1}$

(1) (0, ∞) (2) (– ∞, 0)

(3) (– ∞, + ∞) (4) none of these

32. Find the range of the $f(x)=\frac{x^2}{x^2+1}$

(1) (1, 2) (2) [0, 1)

(3) (0, 1] (4) none of these

33. If $ax^2 + bx + c = 0$ and $bx^2 + cx + a = 0$ have a common root and abc ≠ 0 then = $\frac{a^3+b^3+c^3}{abc}$

(1) 3 (2) 2

(3) 1 (4) 4

34. In solving the equation x2 + ax + b=0 wrong value of 'a' was taken and the roots were found to be 6 and 2. If again a wrong value of b was taken and the roots were found to be –2 and 9 then the correct roots of the original equation are

(1) –3, 4

(2) 3, 4

(3) –7, –2

(4) –3, –4

35. If A + B + C = 180° then cos 2A + cos 2B + cos 2C + 1 =

(1) –4 sin A sin B cos C

(2) –4 cos A cos B sin C

(3) –4 cos A cos B cos C

(4) –4 sin A cos B sin C

36. The principal solution of $\sin\theta = \sqrt{\frac{1+\sqrt{2}}{2\sqrt{2}}}$ is

(1) $22\frac{1}{2}°$ (2) $67\frac{1}{2}°$

(3) 72° (4) 9°

37. The set of values of 'x' for which $\frac{\tan 3x - \tan 2x}{1+\tan 3x \tan 2x} = 1$ is

(1) ϕ (2) $\{\pi/12\}$

(3) $\left\{n\pi + \frac{\pi}{4}; n \in z\right\}$ (4) $\left\{2n\pi + \frac{\pi}{4}; n \in z\right\}$

38. If $81^{\sin^2 x} + 81^{\cos^2 x} = 30, 0 \le x \le \frac{\pi}{2}$ then x =

(1) $\pi/4$ or $\pi/2$ (2) $\pi/6$ or $\pi/3$

(3) $\pi/4$ or $3\pi/4$ (4) $2\pi/3$ or $3\pi/4$

39. If A = {x | x is a multiple of 10 and x ≤ 100} find numbr of elements in powr set of A.

(1) 1027 (2) 512

(3) 2048 (4) 4096

40. If A = {1, 2, 4}, B = {1, 4, 6, 9} and R is a relation from A to B defined by 'x' is greater than 'y'. The range of R is:

(1) {1, 4, 6, 9} (2) {4, 6, 9}

(3) {1} (4) none of these

41. Suppose three distinct non zero real numbers satisfying $a^2 (b + k) = b^2 (b + k) = c^2 (c + k)$ where k is some real number, then value of $\frac{1}{a}+\frac{1}{b}+\frac{1}{c}$ is

(1) 0 (2) k

(3) – k (4) 2k

42. Find sum of n terms of $1 + 2x + 3x^2 + 4x^3 + ...$

(1) $S_n = \frac{(1-x^n)}{(1-x)^2} - \frac{nx^n}{1-x}$ (2) $S_n = \frac{1-x^n}{(1-x)^3} - \frac{nx^n}{1-x}$

(3) $S_n = \frac{1-x^n}{(1-x)^3} - \frac{nx^{n-1}}{1-x}$ (4) $S_n = \frac{1-x^n}{(1-x)^2} - \frac{nx^{n-1}}{(1-x)^3}$

43. If B is the set of numbers obtained by adding 1 to each of the even numbers, then its set builder notation is

(1) B = { x : x is odd and x > 1}

(2) B = { x : x is even)

(3) B = {x : x is odd and x ∈ Z}

(4) B = { x : x is an integer}

44. In a survey of 600 students in a school, 150 students were found to be taking tea and 225 taking coffee, 100 were taking both tea and coffee. Find how many students were taking neither tea nor coffee

(1) 275 (2) 325

(3) 375 (4) 225

45. A line with direction cosines proportional to 2, 1, 2 meets each of the lines x = y + a = z and x + a = 2y = 2z. The co-ordinates of each of the points of intersection are given by :

(1) (3a, 3a, 3a), (a, a, a) (2) (3a, 2a, 3a), (a, a, a)

(3) (3a, 2a, 3a), (a, a, 2a) (4) (2a, 3a, 3a), (2a, a, a)

46. Let A = {1, 2, 3, ..., 45} and R be the relation 'is square of' in A. Which of the following is/are correct?

(a) R ={(1, 1), (4, 2), (9, 3), (16, 4), (25, 5), (36, 6)}

(b) Domain of R = { 1, 4, 9, 16, 25, 36}

(c) Range of R = {1, 2, 3, 4, 5, 6}

(1) a & b (2) b & c

(3) a & c (4) a, b & c

47. Statement I If $|z_1| = |z_2| = |z_3 = ... = |z_n| = 1$, then $|z_1 + z_2 + z_3 + ... + z_n| = \left|\frac{1}{z_1} + \frac{1}{z_2} + \frac{1}{z_3} + ... + \frac{1}{z_n}\right|$.

Statement II If $|z| = 1$, then $z = \frac{1}{\overline{z}}$.

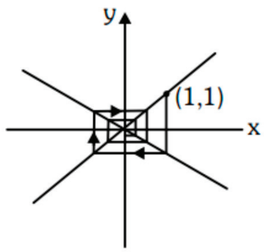
(1) Statement I is correct

(2) Statement II is correct

(3) Both are correct

(4) Neither I nor II is correct

48. Consider two fixed lines y – x = 0 and ay + x = 0, a > 1. A particle P starts from (1, 1) to reach the origin in the manner depicted in the figure. The total distance covered by the particle is



(1) $\frac{a+1}{a-1}$ (2) $\frac{2(a+1)}{a-1}$

(3) $\frac{a-1}{a+1}$ (4) $\frac{2(a-1)}{a+1}$

49. For the index 4, which of the following is not true for all the coefficients?

(1) Coefficients are ${}^4C_0$, ${}^4C_1$, ${}^4C_2$, ${}^4C_3$, ${}^4C_4$

(2) Coefficients are ${}^4C_1$, ${}^4C_2$, ${}^4C_3$, ${}^4C_4$

(3) 1, 4, 6, 4, 1 are the coefficients

(4) Total number of terms are 5

50. By using binomial theorem, (x+ 2)6 is equal to

(1) $x^6+12x^5+60x^4+150x^3+230x^2+192x+64$

(2) $x^5+12x^6+60x^5+160x^4+240x^3+192x^2+62x$

(3) $x^6+12x^5+60x^4+160x^3+240x^2+192x+64$

(4) None of the above

INTERACTIVE SECTION

Student have to solve either left hand side (LHS) questions OR right hand side (RHS) questions. RHS section is for Harvard-MIT Mathematics Tournament (HMMT) enthusiasts. EHF will be conducting math camp (LEVEL-3) to help students prepare for online HMMT participation. The camp will be conducted by retired IIT-Delhi Maths Professors. All expenses will be borne by EHF. Equal preference will be given to students solving either of these sections. More details of online HMMT is available on EHF Website www.eduhealfoundation.org

LHS SECTION

51. Consider the following statements

Statement I If $\frac{1}{8!}+\frac{1}{9!}=\frac{x}{10!}$, then x is equal to 100.

Statement II If $\frac{1}{6!}+\frac{1}{7!}=\frac{x}{8!}$, then x is equal to 56.

Which of the above statement(s) is/are true?

(1) Only I (2) Only II

(3) Both I and II (4) Neither I nor II

RHS SECTION

HMMT

51. The numbers 1 – 10 are written in a circle randomly. Find the expected number of numbers which are at least 2 larger than an adjacent number.

(1) $\frac{17}{3}$

(2) $\frac{16}{3}$

(3) $\frac{15}{2}$

(4) none of these

LHS SECTION

52. If angle θ is divided into two parts such that the tangent of the one part is K times the tangent of the other, and θ is their difference, then sin θ is equal to

(1) $\frac{K+1}{K-1}\sin\phi$

(2) $\frac{K+1}{K-1}\sin\frac{\phi}{2}$

(3) $\frac{K+1}{K-1}\sin\frac{\theta}{2}$

(4) $\frac{K-1}{K+1}\sin\phi$

53. The coefficient of the middle term in the binomial expansion in powers of x of $(1+\alpha x)^4$ and of $(1+\alpha x)^6$ is the same, if α equals

(1) $-\frac{5}{2}$ (2) $\frac{10}{3}$

(3) $-\frac{3}{10}$ (4) $\frac{3}{5}$

54. $\sin x+\sin y=\frac{1}{4}, \sin x-\sin y=\frac{1}{5} \Rightarrow 4\cot\left(\frac{x-y}{2}\right)=$

(1) $5\cot\left(\frac{x-y}{2}\right)$ (2) $5\tan\left(\frac{x-y}{2}\right)$

(3) $5\cot\left(\frac{x+y}{2}\right)$ (4) $5\tan\left(\frac{x+y}{2}\right)$

55. The complex number $\frac{(-\sqrt{3}+3\hat{i})(1-i)}{(3+\sqrt{3}\hat{i})(i)(\sqrt{3}+\sqrt{3}i)}$ when represented in the Argand diagram lies

(1) on the x-axis (real axis)

(2) in the first quadrant

(3) on the y-axis (imaginary axis)

(4) in the second quadrant

RHS SECTION

52. Chris and Paul each rent a different room of a hotel from rooms 1 – 60. However, the hotel manager mistakes them for one person and gives "Chris Paul" a room with Chris's and Paul's room concatenated. For example, if Chris had 15 and Paul had 9, "Chris Paul" has 159. If there are 360 rooms in the hotel, what is the probability that "Chris Paul" has a valid room ?

(1) $\frac{153}{1180}$ (2) $\frac{152}{1180}$

(3) $\frac{150}{1180}$ (4) none of these

53. The numbers 1, 2...11 are arranged in a line from left to right in a random order. It is observed that the middle number is larger than exactly one number to its left. Find the probability that it is larger than exactly one number to its right.

(1) $\frac{10}{45}$ (2) $\frac{10}{35}$

(3) $\frac{10}{33}$ (4) $\frac{10}{28}$

54. Let S = {1, 2...2016}, and let f be a randomly chosen bijection from S to itself. Let n be the smallest positive integer such that $f^{(n)}(1) = 1$, where $f^{(i)}(x) = f(f^{(i-1)}(x))$. What is the expected value of n?

(1) $\frac{2016}{2}$ (2) $\frac{2018}{3}$

(3) $\frac{2017}{2}$ (4) none of these

55. Let the sequence ai be defined as $a_{i+1} = 2^{a_i}$...Find the number of integers $1 \le n \le 1000$ such that if $a_0 = n$, then 100 divides $a_{1000} - a_1$.

(1) 40 (2) 30

(3) 20 (4) 50

LHS SECTION

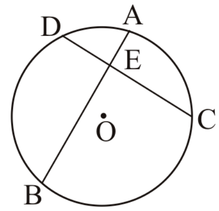
56. A, B, C, D lie on circle such that chord BC subtend 110º at centre O of circle, then ∠BEC is

(1) 40º

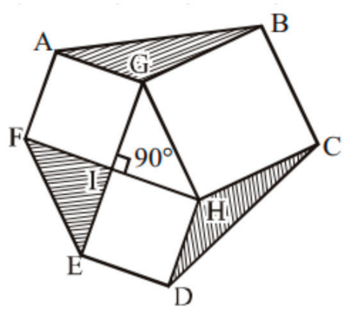
(2) 70º

(3) 80º

(4) 55º



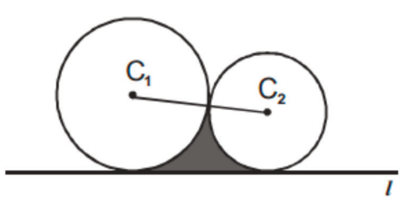
57. As shown in figure hexagon ABCDEFA is divided into 3 squares and 4 triangles such that sum of areas of shaded region is 6 sq. units, side and length AG & HD are a & b units respectively. If a & b are roots of equation $x^2 + (2k - k^2)x + (2k - 6) = 0$, where $k \in R$, then value of $(\alpha + \beta)$ is



(1) 10 (2) 8

(3) 15 (4) 24

58. The perimeter of the shaded region of the given figure is (where radii of two externally touching circles with centres C_1 & C_2 are $\frac{3}{2}$ & $\frac{1}{2}$ respectively and having common tangent *l*)



(1) $\frac{5\pi}{6}+\sqrt{3}$ (2) $\frac{2\pi}{3}+\sqrt{3}$

(3) $\pi-\sqrt{3}$ (4) $\pi+\frac{\sqrt{3}}{2}$

RHS SECTION

56. Meghal is playing a game with 2016 rounds 1, 2,.... 2016. In round n, two rectangular double-sided mirrors are arranged such that they share a common edge and the angle between the faces is $\frac{2\pi}{n+2}$. Meghal shoots a laser at these mirrors and her score for the round is the number of points on the two mirrors at which the laser beam touches a mirror. What is the maximum possible score Meghal could have after she finishes the game?

(1) 2018088 (2) 1019088

(3) 1019003 (4) 1015086

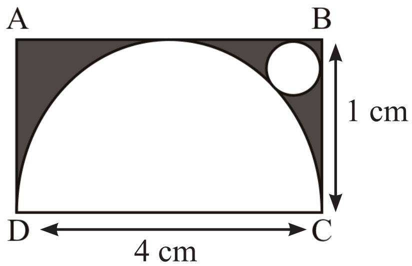
57. Let ABC be a triangle with AB = 5, BC = 6, and AC = 7. Let its orthocenter be H and the feet of the altitudes from A, B, C to the opposite sides be D, E, F respectively. Let the line DF intersect the circumcircle of AHF again at X. Find the length of EX.

(1) $\frac{180}{48}$ (2) $\frac{190}{32}$

(3) $\frac{190}{49}$ (4) $\frac{190}{42}$

58. Alex has an 20 × 16 grid of lightbulbs, initially all off. He has 36 switches, one for each row and colum. Flipping the switch for the ith row will toggle the state of each lightbulb in the ith row (so that if it were on before, it would be off, and vice versa). Similarly, the switch for the jth column will toggle the state of each bulb in the jth column. Alex makes some (possibly empty) sequence of switch flips, resulting in some configuration of the lightbulbs and their states. How many distinct possible configurations of lightbulbs can Alex achieve with such a sequence? Two configurations are distinct if

59. The figure shows a rectangle ABCD with dimensions 2 cm × 1 cm. A semi-circle and a circle inscribed inside it as shown. What is the ratio of the area of the circle to the semicircle



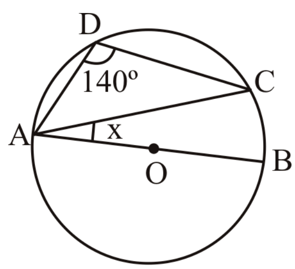
(1) $\left(\sqrt{2}-1\right)^2$

(2) $\left(\sqrt{2}-1\right)^2/1$

(3) $2\left(\sqrt{2}-1\right)^2$

(4) None of these

60. In the figure, O is the centre of the circle. If ∠ADC = 140°, then what is the value of x?



(1) 35°
(2) 55°
(3) 60°
(4) 50°

there exists a lightbulb that is on in one configuration and off in another.

(1) 2^{35}
(2) 4^{35}
(3) 3^{45}
(4) none of these

59. A cylinder with radius 15 and height 16 is inscribed in a sphere. Three congruent smaller spheres of radius x are externally tangent to the base of the cylinder, externally tangent to each other, and internally tangent to the large sphere. What is the value of x?

(1) $\dfrac{15\sqrt{35}-73}{4}$
(2) $\dfrac{15\sqrt{45}-70}{4}$
(3) $\dfrac{15\sqrt{40}-70}{3}$
(4) $\dfrac{15\sqrt{37}-75}{4}$

60. Determine the largest integer n such that there exist monic quadratic polynomials $p_1(x)$, $p_2(x)$, $p_3(x)$ with integer coefficients so that for all integers $i \in [1, n]$ there exists some $j \in [1, 3]$ and $m \in Z$ such that $p_j(m) = i$

(1) 11

(2) 10

(3) 7

(4) 9

END OF THE EXAM